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# On the Consistency of Orbifolds

Oren Bergman\*

*California Institute of Technology, Pasadena, CA 91125, USA*  
 and

*CIT/USC Center for Theoretical Physics, Univ. of Southern California, Los Angeles CA*

Matthias R. Gaberdiel†

*Department of Applied Mathematics and Theoretical Physics*  
*Centre for Mathematical Sciences*  
*Wilberforce Road*  
*Cambridge CB3 0WA, U.K.*

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## Abstract

Modular invariance is a necessary condition for the consistency of any closed string theory. In particular, it imposes stringent constraints on the spectrum of orbifold theories, and in principle determines their spectrum uniquely up to discrete torsion classes. In practice, however, there are often ambiguities in the construction of orbifolds that are a consequence of the fact that the action of the orbifold elements on degenerate ground states is not unambiguous. We explain that there exists an additional consistency condition, related to the spectrum of D-branes in the theory, which eliminates these ambiguities. For supersymmetric orbifolds this condition turns out to be equivalent to the condition that supersymmetry is unbroken in the twisted sectors, but for non-supersymmetric orbifolds it appears to be a genuinely new consistency condition.

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\*E-mail address: [bergman@theory.caltech.edu](mailto:bergman@theory.caltech.edu)

†E-mail address: [M.R.Gaberdiel@damtp.cam.ac.uk](mailto:M.R.Gaberdiel@damtp.cam.ac.uk)

**1. Introduction.** The orbifold construction is one of the most powerful techniques in string theory that allows one to obtain interesting new theories with reduced symmetries [1]. This procedure can be applied whenever a string theory possesses a discrete symmetry group  $\Gamma$ . The construction proceeds in two steps: first we restrict the space of states to those that are invariant under  $\Gamma$ , and then we add suitably projected twisted sectors, as needed by modular invariance. The initial projection is equivalent to adding sectors to the partition function in which the boundary condition in the timelike world-sheet direction  $t$  is twisted by generators of  $\Gamma$ . One of the generators of the modular group of the torus,  $S : \tau \mapsto -1/\tau$ , exchanges the timelike direction  $t$  with the spacelike world-sheet direction  $\sigma$ . In order for the partition function to be modular invariant, the theory must therefore also contain sectors in which the boundary condition in  $\sigma$  is twisted by the generators of  $\Gamma$ ; these are the twisted sectors. Finally, invariance under the other generator of the modular group,  $T : \tau \mapsto \tau + 1$ , requires us to add sectors in which both boundary conditions are twisted; in essence this amounts to projecting the twisted sectors by  $\Gamma$  as well.

Whilst this algorithm determines the theory in principle, there are a number of ambiguities that are not fixed by modular invariance of the one-loop vacuum amplitude alone. For example, if we are dealing with a string theory in the RNS formalism, every sector of the theory must be GSO-projected. In each sector that possesses fermionic zero modes, and therefore a degenerate ground state, the GSO-projection acts as the chirality operator on the ground state. *A priori*, the definition of this operator is only fixed up to a sign, and this sign choice affects the spectrum of the theory significantly. On the other hand, the partition function is insensitive to this choice, and therefore the condition of one-loop vacuum modular invariance does not fix this ambiguity.

It is generally believed that these ambiguities are resolved by the conditions that come from non-vacuum and higher-loop amplitudes, and that the remaining freedom is described in terms of discrete torsion [2]. In practice, it is however difficult to determine the consistent GSO-projections explicitly. In fact, the analysis of [2] describes the freedom in modifying the action of the orbifold (and GSO-projections) in the twisted sectors *relative* to a certain solution which is assumed to be consistent, namely the one in which all phases are taken to be +1. In particular, this requires that the above sign ambiguities have been chosen so as to give a consistent solution, but it is *a priori* not obvious how this should be done.

As we shall explain in this paper, there exists a simple *non-perturbative* consistency condition that fixes these ambiguities, at least for a certain class of theories, uniquely. This condition arises from a careful analysis of the D-brane spectrum of these theories. The D-brane spectrum can be determined using the boundary state formalism [3, 4, 5, 6, 7, 8], in which D-branes are described by coherent states of the closed string sector. Once the various projection operators of the closed string theory have been fixed, the D-brane spectrum is uniquely determined. However, not every such spectrum is allowed, since there exist transitions between different D-brane configurations. Specifically, when a D-brane in the bulk collides with a fixed-plane of the orbifold it must be allowed to break into fractional D-branes. In order for this to occur, however, both types of branes must exist in the spectrum. As we shall see, this requirement fixes the ambiguities in the GSO-projection of the twisted sectors uniquely.

For orbifold projections that preserve supersymmetry, these ambiguities can also be fixed by requiring the twisted sectors to preserve the same supersymmetry. This is the case, for example, for Type IIA or IIB on  $T^4/\mathbb{Z}_2$ , where the  $\mathbb{Z}_2$  generator acts by reflection of the four compact coordinates. In these cases, the supersymmetry considerations pick out precisely the theory that is also non-perturbatively consistent. For orbifolds that break supersymmetry completely, however, there does not appear to be an analogous perturbative criterion that selects the non-perturbatively consistent theory.

Our interest in this problem arose from recent work of Klebanov, Nekrasov and Shatashvili [16], who considered Type IIB on  $T^6/\mathbb{Z}_4$ , where the  $\mathbb{Z}_4$  generator acts by the reflection of the six compact directions. This orbifold breaks all the supersymmetries of Type IIB, and is in fact equivalent to Type 0B on  $T^6/\mathbb{Z}_2$ . As we shall discuss in detail below, their choice for the GSO-projection in the twisted sectors does not satisfy the above condition, and therefore does not define a (non-perturbatively) consistent theory. This resolves the question of which state is charged under the  $U(1)$  gauge field that comes from the twisted R-R sector, since the consistent theory does not have such a gauge field.

**2. D-branes on  $\mathbb{Z}_2$  orbifolds.** Let us begin by describing briefly the D-brane spectrum of a  $\mathbb{Z}_2$  orbifold, where the  $\mathbb{Z}_2$  generator acts by the inversion of  $n$  coordinates. (More details can be found in [8]; see also [9, 10, 11, 12, 13, 14, 15].) For simplicity we shall consider the non-compact case  $\mathbb{R}^n/\mathbb{Z}_2$ , which only has one fixed plane (of spatial dimension  $9 - n$ ) at the origin.<sup>1</sup> There exist three possible types of D-branes, which differ in their boundary state components. **Bulk** D-branes have components only in the untwisted sectors,

$$|Dp\rangle_b = \left( |Bp\rangle_{\text{NS-NS};U} + |Bp\rangle_{\text{R-R};U} \right), \quad (1)$$

**fractional** D-branes have components in all untwisted and twisted sectors,

$$|Dp\rangle_f = \frac{1}{2} \left( |Bp\rangle_{\text{NS-NS};U} + |Bp\rangle_{\text{R-R};U} + |Bp\rangle_{\text{NS-NS};T} + |Bp\rangle_{\text{R-R};T} \right), \quad (2)$$

and **truncated** D-branes only involve the untwisted NS-NS and twisted R-R sectors,

$$|Dp\rangle_t = \frac{1}{\sqrt{2}} \left( |Bp\rangle_{\text{NS-NS};U} + |Bp\rangle_{\text{R-R};T} \right), \quad (3)$$

and can only exist for values of  $p$  for which a fractional (and bulk) D-brane does not exist. The above spectrum, including the latter restriction, follows from the usual condition that the cylinder amplitude between any two D-branes must correspond to an open string partition function.

In a supersymmetric theory (1) and (2) are BPS states, whereas (3) are non-BPS, but nevertheless stable in certain regimes of the moduli space [12, 13]. To account for the different orientations of the D-branes relative to the action of the orbifold, we denote as in [8] the number

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<sup>1</sup>If  $m$  of the  $n$  directions on which the  $\mathbb{Z}_2$  acts are compact there will be  $2^m$  fixed planes.

of world-volume directions parallel to the fixed-plane by  $r$ , and the number of world-volume directions transverse to the fixed-plane by  $s$ . We shall henceforth label the  $Dp$ -branes (and boundary states) by  $(r, s)$ , where  $p = r + s$ . Fractional and truncated branes are either completely localised at the fixed-plane (if  $s = 0$ ), or extend in  $s$  directions transverse to the fixed-plane and terminate at an  $r$ -dimensional hyperplane in it; this follows from the fact that the boundary states have components in the twisted sectors.<sup>2</sup>

Physical closed string states must be invariant under the GSO-projection and the action of the orbifold group. The spectrum of *physical* D-branes, *i.e.* the allowed values of  $r$  and  $s$  for the three types of branes, is therefore determined by the action of the GSO and orbifold operators on the boundary states in the four different sectors. Since these boundary states are generically given by

$$|B(r, s)\rangle = e^{\sum (\pm \frac{1}{n} \alpha_{-n}^i \tilde{\alpha}_{-n}^i \pm \psi_{-r}^i \tilde{\psi}_{-r}^i)} |B(r, s)\rangle^{(0)}, \quad (4)$$

the crucial constraint comes from the action of the above operators on the ground states of the different sectors. The left- and right-moving GSO-operators are given as

$$(-1)^f = \pm (i^{|\mathcal{F}|/2}) \prod_{\mu \in \mathcal{F}} (\sqrt{2} \psi_0^\mu) \quad (-1)^{\tilde{f}} = \pm (i^{|\mathcal{F}|/2}) \prod_{\mu \in \mathcal{F}} (\sqrt{2} \tilde{\psi}_0^\mu), \quad (5)$$

where  $\psi_0^\mu$  and  $\tilde{\psi}_0^\mu$  are the left- and right-moving fermionic zero modes, respectively, which satisfy the Clifford algebra

$$\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu} \quad \{\psi_0^\mu, \tilde{\psi}_0^\nu\} = 0 \quad \{\tilde{\psi}_0^\mu, \tilde{\psi}_0^\nu\} = \eta^{\mu\nu}, \quad (6)$$

and  $\mathcal{F}$  denotes the set of coordinates in which the given sector has fermionic zero modes. The prefactor has been fixed (up to a sign) so that both operators square to the identity (we shall only consider the case where the number of elements in  $\mathcal{F}$ ,  $|\mathcal{F}|$ , is even). In the untwisted NS-NS sector, for which  $|\mathcal{F}| = 0$ , it is conventional to take both signs to be  $-$ , so that the tachyonic ground state is odd under both operators. For definiteness we adopt this convention for all other sectors as well. Finally, the inversion operator of the orbifold acts as

$$g = \prod_{\mu \in \mathcal{F} \cap \mathcal{R}} (\sqrt{2} \psi_0^\mu) \prod_{\mu \in \mathcal{F} \cap \mathcal{R}} (\sqrt{2} \tilde{\psi}_0^\mu), \quad (7)$$

where  $\mathcal{R}$  denotes the set of coordinates on which the orbifold acts. Again, the definition of  $g$  in the twisted sectors is *a priori* ambiguous up to a sign; for definiteness we have again fixed this to be  $+$  in all sectors. The resulting actions on the boundary states<sup>3</sup> are shown in table 1 (see [8] for details).

The definition of the GSO and orbifold projections in the various sectors determines which of the boundary states are physical, and therefore the D-brane spectrum of the theory. However,

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<sup>2</sup>In the compact case fractional and truncated branes would contain components in multiple twisted sectors, and would therefore be suspended between different fixed planes.

<sup>3</sup>One must actually combine boundary states with different spin structures to get GSO-eigenstates; the listed states are the linear combination for which  $(-1)^f$  has eigenvalue  $+1$ .

boundary state	$(-1)^f$	$(-1)^{\tilde{f}}$	$g$
$ B(r, s)\rangle_{\text{NS-NS}; \text{U}}$	+1	+1	+1
$ B(r, s)\rangle_{\text{R-R}; \text{U}}$	+1	$(-1)^{r+s+1}$	$(-1)^s$
$ B(r, s)\rangle_{\text{NS-NS}; \text{T}}$	+1	$i^n (-1)^s$	$(-1)^s$
$ B(r, s)\rangle_{\text{R-R}; \text{T}}$	+1	$i^n (-1)^{r+1}$	+1

Table 1: GSO and orbifold actions on the boundary states.

in order for this D-brane spectrum to make sense, it must satisfy an additional consistency condition: as a bulk  $D(r, s)$ -brane approaches the fixed plane, additional massless scalars usually appear in the world-volume gauge theory; these parametrise the Coulomb branch, and describe the moduli along which the bulk brane fractionates into two fractional branes. In order for this to be possible, the theory must therefore also have a fractional  $D(r, s)$ -brane. As we shall see below, this condition fixes uniquely the ambiguity in defining the GSO-projection in the twisted sectors.

We shall illustrate this condition by considering two examples. The first is Type II on  $\mathbb{R}^4/\mathbb{Z}_2$ , which is supersymmetric, and the second is Type 0 on  $\mathbb{R}^6/\mathbb{Z}_2$  (or equivalently Type II on  $\mathbb{R}^6/\mathbb{Z}_4$ ), in which supersymmetry is broken.

**3. Supersymmetric example.** Consider Type II strings on  $\mathbb{R}^{1,5} \times \mathbb{R}^4/\mathbb{Z}_2$ , where the generator  $g$  of  $\mathbb{Z}_2$  reflects the coordinates  $x^5, x^6, x^7, x^8$ . This breaks  $SO(1, 9)$  to  $SO(1, 5) \times SO(4)_R$  ( $SO(4)_S \times SO(4)_R$  in light-cone gauge), where the second factor corresponds to a global R-symmetry from the six-dimensional point of view. For Type IIA the resulting six-dimensional theory has  $\mathcal{N} = (1, 1)$  supersymmetry, and for Type IIB it has  $\mathcal{N} = (2, 0)$  supersymmetry. The relevant properties of the different sectors are shown in table 2.

sector	$\alpha_n^i$	$\psi_r^i$	$SO(4)_S \times SO(4)_R$
NS;U	$n \in \mathbb{Z}$	$r \in \mathbb{Z} + 1/2$	$(\mathbf{1}, \mathbf{1})$
R;U	$n \in \mathbb{Z}$	$r \in \mathbb{Z}$	$(\mathbf{2} \oplus \mathbf{2}', \mathbf{2} \oplus \mathbf{2}')$
NS;T	$n \in \begin{cases} \mathbb{Z} & i = 1, \dots, 4 \\ \mathbb{Z} + 1/2 & i = 5, \dots, 8 \end{cases}$	$r \in \begin{cases} \mathbb{Z} + 1/2 & i = 1, \dots, 4 \\ \mathbb{Z} & i = 5, \dots, 8 \end{cases}$	$(\mathbf{1}, \mathbf{2} \oplus \mathbf{2}')$
R;T	$n \in \begin{cases} \mathbb{Z} & i = 1, \dots, 4 \\ \mathbb{Z} + 1/2 & i = 5, \dots, 8 \end{cases}$	$r \in \begin{cases} \mathbb{Z} & i = 1, \dots, 4 \\ \mathbb{Z} + 1/2 & i = 5, \dots, 8 \end{cases}$	$(\mathbf{2} \oplus \mathbf{2}', \mathbf{1})$

Table 2: Oscillator modings in light-cone gauge and ground state charges of the different sectors in the orbifold  $\mathbb{R}^4/\mathbb{Z}_2$ . The ground state is tachyonic in the NS;U sector, and massless in all the other sectors. Its charges are determined by quantizing the fermionic zero modes.

Modular invariance requires that we project all sectors onto states which are even under  $g$ ,

$$P_{orbifold} = \frac{1}{2}(1 + g) . \quad (8)$$

In principle, modular invariance should also determine the correct GSO-projections in the twisted sectors, once that of the untwisted R-R sector has been fixed, *i.e.* once we have decided whether to start with Type IIA or Type IIB string theory. However, due to the sign ambiguity in the action of the GSO operators on the ground states of these sectors, it is actually difficult to determine the consistent projections. The most general GSO-projections are given as<sup>4</sup>:

$$P_{GSO} = \begin{cases} \frac{1}{4}(1 + (-1)^f)(1 + (-1)^{\tilde{f}}) & \text{NS-NS;U} \\ \frac{1}{4}(1 + (-1)^f)(1 + \epsilon(-1)^{\tilde{f}}) & \text{R-R;U} \\ \frac{1}{4}(1 + (-1)^f)(1 + \eta(-1)^{\tilde{f}}) & \text{NS-NS;T} \\ \frac{1}{4}(1 + (-1)^f)(1 + \delta(-1)^{\tilde{f}}) & \text{R-R;T}, \end{cases} \quad (9)$$

where  $\epsilon, \eta, \delta = \pm 1$ . The phase  $\epsilon$  corresponds to whether the theory is Type IIA ( $\epsilon = -1$ ) or Type IIB ( $\epsilon = +1$ ), and the phases  $\eta, \delta$  correspond to the aforementioned ambiguity.

To fix the ambiguity in the choice for  $\eta$  and  $\delta$ , let us now consider the D-brane spectrum of the theory. From the projections in the untwisted sectors it follows that the physical bulk D-branes have  $r$  even and  $s$  even in Type IIA, and  $r$  odd and  $s$  even in Type IIB. In order to satisfy the above fractionation condition, there must exist fractional D-branes for the same values of  $r$  and  $s$ . In particular, this means that the corresponding boundary states must be physical in both the twisted NS-NS and the twisted R-R sectors. It follows from the results of Table 1 that this requires  $\eta = +1$  and  $\delta = \eta\epsilon = \epsilon$ ; the D-brane spectrum can then be summarised by

- (i) **IIA on  $\mathbb{R}^4/\mathbb{Z}_2$** : Fractional and bulk D-branes exist for  $r$  and  $s$  both even.  
Truncated D-branes exist for  $r$  even and  $s$  odd, *e.g.* the non-BPS D-string [12, 13].
- (ii) **IIB on  $\mathbb{R}^4/\mathbb{Z}_2$** : Fractional and bulk D-branes exist for  $r$  odd and  $s$  even.  
Truncated D-branes exist for both  $r$  and  $s$  odd.

This choice of  $\eta$  and  $\delta$  also leads to a supersymmetric spectrum in the twisted sectors. Indeed, the surviving components of the ground state in the twisted R-R sector transform as

$$(\mathbf{2}, \mathbf{1}) \otimes (\mathbf{2}', \mathbf{1}) = (\mathbf{4}, \mathbf{1}) \quad (10)$$

in the Type IIA orbifold, and as

$$(\mathbf{2}, \mathbf{1}) \otimes (\mathbf{2}, \mathbf{1}) = (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}) \quad (11)$$

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<sup>4</sup>Only the relative phase of  $(-1)^f$  and  $(-1)^{\tilde{f}}$  is relevant for our purposes

in the Type IIB orbifold. The former corresponds to the vector component of an  $\mathcal{N} = (1, 1)$  vector multiplet, and the latter to the rank two antisymmetric tensor component and one of the scalar components of an  $\mathcal{N} = (2, 0)$  tensor multiplet. As these are precisely the unbroken supersymmetries associated with the respective theories, the above choice for the GSO-projection in the twisted sectors is consistent with supersymmetry as well.

It may also be worth mentioning that only this choice for the GSO-projection in the twisted sectors leads to an orbifold that can be blown up to a smooth ALE space (K3 in the fully compact case), since the latter is manifestly supersymmetric.

**4. Non-supersymmetric example.** Now consider the case of Type 0 strings on  $\mathbb{R}^{1,3} \times \mathbb{R}^6/\mathbb{Z}_2$ , where the generator  $g$  of  $\mathbb{Z}_2$  reflects the six coordinates  $x^3, \dots, x^8$ .<sup>5</sup> Note that  $g^2 = (-1)^F$ , but since Type 0 is purely bosonic this is equivalent to the identity. In Type II strings  $g$  generates a  $\mathbb{Z}_4$  group, so the above theory is identical to Type II on  $\mathbb{R}^{1,3} \times \mathbb{R}^6/\mathbb{Z}_4$ . The ten-dimensional Lorentz group is broken to  $SO(1, 3) \times SO(6)$  ( $SO(2) \times SO(6)$  in light-cone gauge), and the theory is not supersymmetric (and in fact completely fermion-free). The different sectors of the theory are described in table 3.

sector	$\alpha_n^i$	$\psi_r^i$	$SO(2)_S \times SO(6)_R$
NS;U	$n \in \mathbb{Z}$	$r \in \mathbb{Z} + 1/2$	$\mathbf{1}_0$
R;U	$n \in \mathbb{Z}$	$r \in \mathbb{Z}$	$(\mathbf{4} \oplus \overline{\mathbf{4}})_{\frac{1}{2}} \oplus (\mathbf{4} \oplus \overline{\mathbf{4}})_{-\frac{1}{2}}$
NS;T	$n \in \begin{cases} \mathbb{Z} & i = 1, 2 \\ \mathbb{Z} + 1/2 & i = 3, \dots, 8 \end{cases}$	$r \in \begin{cases} \mathbb{Z} + 1/2 & i = 1, 2 \\ \mathbb{Z} & i = 3, \dots, 8 \end{cases}$	$\mathbf{4}_0 \oplus \overline{\mathbf{4}}_0$
R;T	$n \in \begin{cases} \mathbb{Z} & i = 1, 2 \\ \mathbb{Z} + 1/2 & i = 3, \dots, 8 \end{cases}$	$r \in \begin{cases} \mathbb{Z} & i = 1, 2 \\ \mathbb{Z} + 1/2 & i = 3, \dots, 8 \end{cases}$	$\mathbf{1}_{\frac{1}{2}} \oplus \mathbf{1}_{-\frac{1}{2}}$

Table 3: Oscillator modings and ground state charges of the different sectors in the orbifold  $\mathbb{R}^6/\mathbb{Z}_2$ . The ground state is tachyonic in the NS;U sector, massive in the NS;T sector, and massless in the R;U and R;T sectors.

Since we are using the Type 0 picture, the relevant GSO operator is actually the combination  $(-1)^{f+\tilde{f}}$ . But since the action of  $(-1)^f$  is trivial on all boundary states, we can still refer to table 1 (with  $n = 6$  in this case) for the transformation properties of the boundary states.<sup>6</sup> The

<sup>5</sup>This case is of particular interest since it is directly related to recent work of Klebanov, Nekrasov and Shatashvili [16], where this orbifold of Type 0B was discussed.

<sup>6</sup>In fact, both  $(-1)^f$  and  $(-1)^{\tilde{f}}$  change the spin structure of the boundary states, and the Type 0 GSO operator  $(-1)^{f+\tilde{f}}$  therefore preserves the spin structure. Thus it is not necessary to consider a linear combination of boundary states, and this gives rise to the famous doubling of the D-brane spectrum of Type 0 relative to Type II.

most general GSO-projections are now given by

$$P_{GSO} = \begin{cases} \frac{1}{2}(1 + (-1)^{f+\tilde{f}}) & \text{NS-NS} \\ \frac{1}{2}(1 + \epsilon(-1)^{f+\tilde{f}}) & \text{R-R} \\ \frac{1}{2}(1 + \eta(-1)^{f+\tilde{f}}) & \text{NS-NS;T} \\ \frac{1}{2}(1 + \delta(-1)^{f+\tilde{f}}) & \text{R-R;T} , \end{cases} \quad (12)$$

where  $\epsilon$  again determines whether the theory is Type A or B.

Referring again to table 1, it now follows that in order to satisfy the D-brane fractionation condition, we must have  $\eta = -1$ , and  $\delta = \eta\epsilon = -\epsilon$ . For this choice (and only for this choice) we obtain a spectrum of D-branes that is consistent with the above fractionation condition; this D-brane spectrum can be summarised by

- (i) **0A on  $\mathbb{R}^6/\mathbb{Z}_2$** : Fractional and bulk D-branes exist for  $r$  and  $s$  both even.  
Truncated D-branes exist for  $r$  even and  $s$  odd.
- (ii) **0B on  $\mathbb{R}^6/\mathbb{Z}_2$** : Fractional and bulk D-branes exist for  $r$  odd and  $s$  even.  
Truncated D-branes exist for both  $r$  and  $s$  odd.

The above solutions for  $\eta$  and  $\delta$  are somewhat counterintuitive, in that the GSO-projection in the twisted sectors is opposite to that in the untwisted sectors. In particular, the projection in the twisted R-R sector is *chiral* in Type 0A and *non-chiral* in Type 0B, which is opposite to the convention in the untwisted R-R sector. The surviving massless fields in the twisted R-R sector therefore transform as

$$(\mathbf{1}_{\frac{1}{2}} \otimes \mathbf{1}_{\frac{1}{2}}) \oplus (\mathbf{1}_{-\frac{1}{2}} \otimes \mathbf{1}_{-\frac{1}{2}}) = \mathbf{1}_1 \oplus \mathbf{1}_{-1} \quad (13)$$

in the Type 0A orbifold, and as

$$(\mathbf{1}_{\frac{1}{2}} \otimes \mathbf{1}_{-\frac{1}{2}}) \oplus (\mathbf{1}_{-\frac{1}{2}} \otimes \mathbf{1}_{\frac{1}{2}}) = 2 \times \mathbf{1}_0 \quad (14)$$

in the Type 0B orbifold. The former corresponds to the two helicities of a massless vector in four dimensions, and the latter to two massless scalars.<sup>7</sup>

This is the opposite of what was claimed in [16], namely that the twisted sector of the Type 0B orbifold contains a massless vector. In effect, the authors of [16] chose  $\eta = \delta = +1$ , and therefore a chiral GSO-projection in the twisted R-R sector. It is clear from table 1 that in this case there are no physical fractional branes, and therefore that the fractionation condition is violated.

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<sup>7</sup>For the case of the Type 0B orbifold, this also agrees with the convention that was considered in [17].



**5. Conclusions.** The condition of modular invariance, which underlies all consistent closed string theories, is sometimes clouded by phase ambiguities. This is especially true for the GSO-projection in sectors containing fermionic zero modes. In this note we have explained that there exists a non-perturbative consistency condition for orbifold theories, related to the spectrum of D-branes, that determines the GSO-projection in the twisted sectors uniquely. For the supersymmetric cases, this condition reproduces the conventions that follow from supersymmetry, but it also applies to situations where supersymmetry is broken. As an example of the latter case, we considered Type 0 strings on  $\mathbb{R}^{1,3} \times \mathbb{R}^6/\mathbb{Z}_2$ . In this case, requiring the spectrum of fractional D-branes to match up with that of the bulk D-branes fixes the GSO-projection in the twisted sectors, and thus the spectrum of the theory.

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